The FE^2 method

Multiscale hydro-mechanical modelling in the Lagamine code

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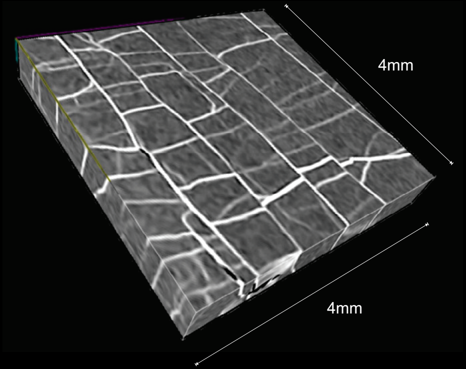
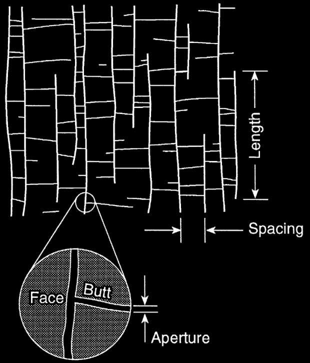
1. Why is multiscale so popular? The example of coal
2. Mind the gap! A brief history of homogenization
3. Lagamine in action

Why is multiscale so popular? The example of coal

Coal is a naturally fractured rock (with a regular pattern).

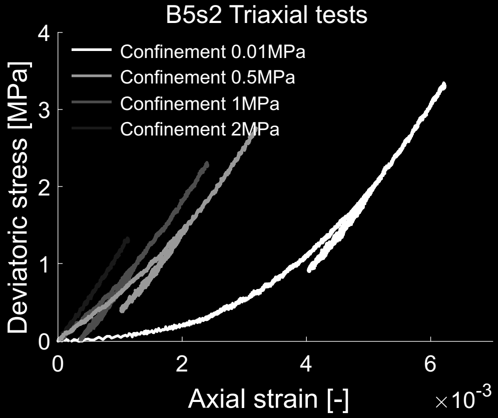


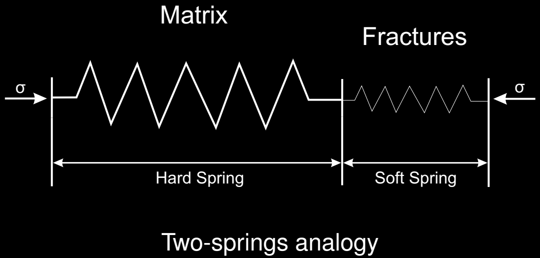
Tomography imaging [Jing et al., 2016]



[Laubach et al., 1998]

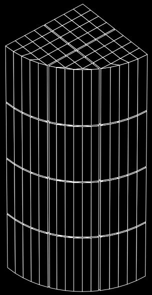
Matrix and fractures behave differently.





Direct modelling:

- **Matrix**: simple isotropic elastic law (e.g. E_m and ν_m)
- **Fractures**: hyperbolic normal stiffness evolution with the aperture



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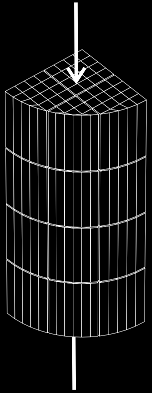

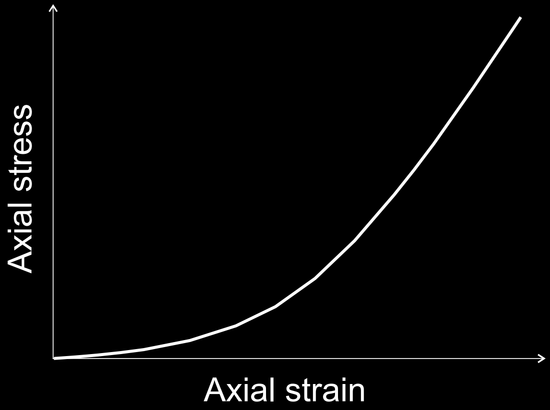

It leads to a global non-linear behaviour of the material.

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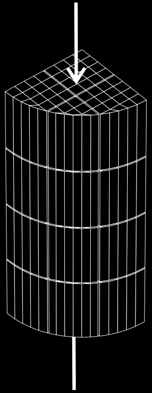
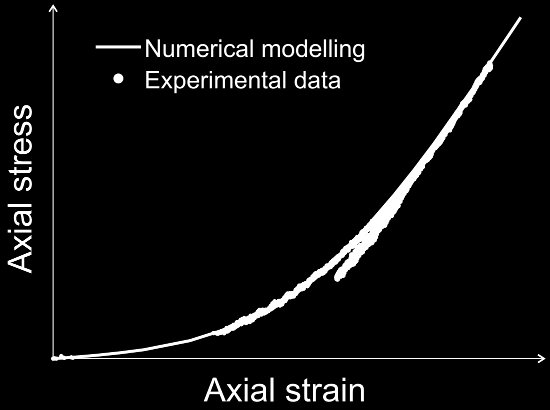
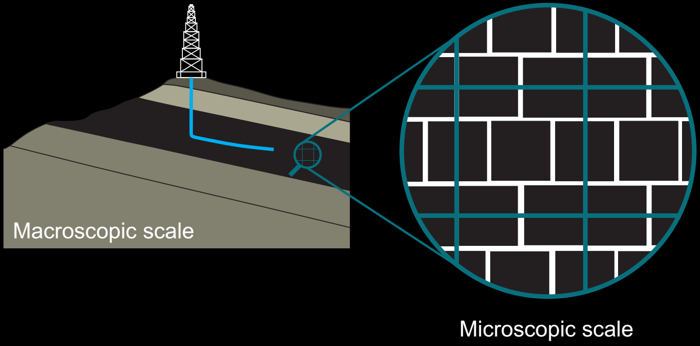




It fits the experimental data with well-chosen parameters.

But **direct modelling** of the whole microstructure is **not possible** for large-scale modelling!

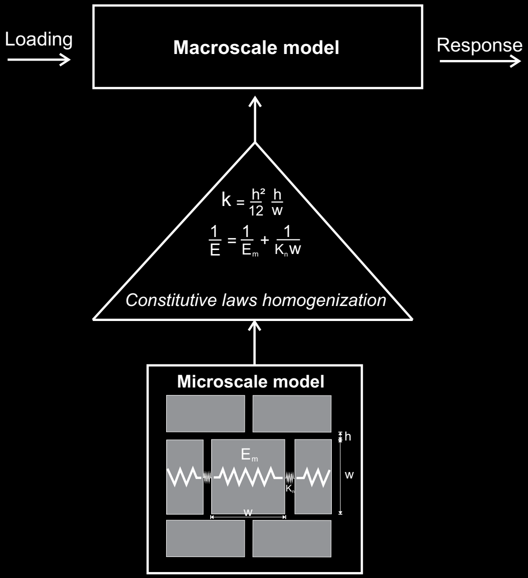


We have a **microscale** constitutive **model** but we want to perform **macroscale modelling**

$\longrightarrow$ **Multiscale** scheme

Mind the gap! A brief history of homogenization

First approach is the homogenization of the laws.



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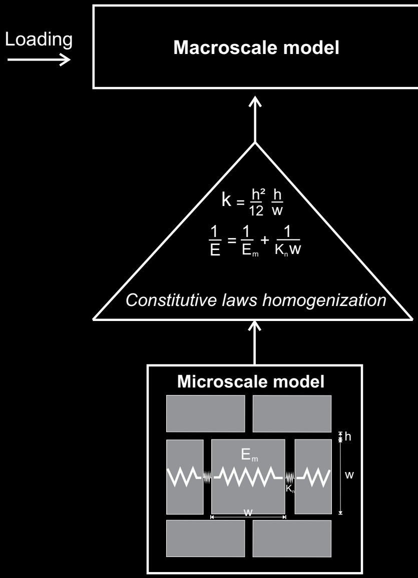

$$[A^M]\,\{\delta U^M\} = \{\delta \Sigma^M\}$$

Macroscale stiffness matrix
based on macroscopic laws

$\{\delta U^M\}$: Nodal displacement components

$\{\delta \Sigma^M\}$: Nodal force components

Exploitation of the microstructure potential is very limited.

Second approach

$$[A^M]\,\{\delta U^M\} = \{\delta \Sigma^M\}$$

Macroscale stiffness matrix
~~based on macroscopic laws~~

$\{\delta U^M\}$: Nodal displacement components

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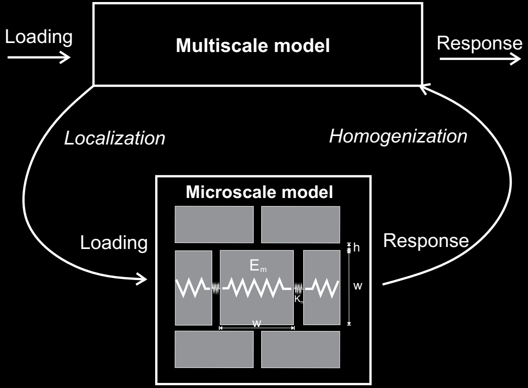
Second approach is the homogenization of the REV response.



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Macroscale stiffness matrix
obtained by assigning a REV to each macro IP

$\{\delta U^M\}$: Nodal displacement components

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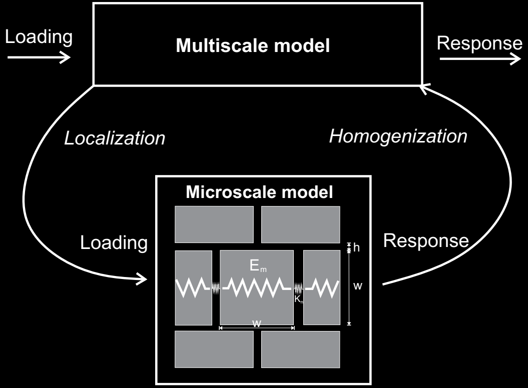
Second approach is the homogenization of the REV response.

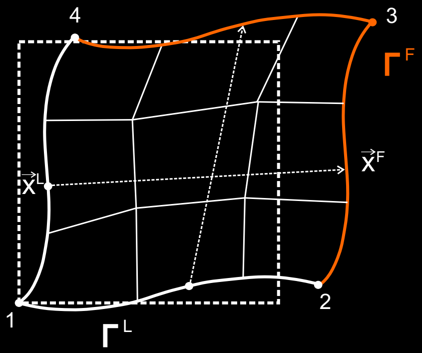


There are 4 main steps:

1. **Localization** of the macroscale deformations on the microscale
2. Resolution of the boundary value problem (**BVP**) on the microscale
3. **Homogenization** of the microscale stresses to compute the macroscopic quantity
4. Resolution of the boundary value problem on the macroscale

⇨ Multi-scale computation

- **Localization** (macro to micro) is performed through the **boundary conditions** (BC).



Periodic BC:

- Displacement:

$$u_i^F = u_i^L + \varepsilon_{ij}^M \left(x_j{}^F - x_j{}^L\right)$$

- Water pressure:

$$p_w^F = p_w^L + (\nabla p_w)_x^M \left(x_j{}^F - x_j{}^L\right)$$

- Gas pressure:

$$p_g^F = p_g^L + (\nabla p_g)_x^M \left(x_j{}^F - x_j{}^L\right)$$

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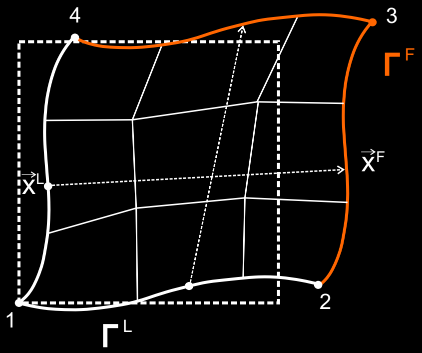

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Separation of scales requirement!

$$l_{REV} \ll \frac{p^M}{\nabla p^M}$$

2. The mechanical microscale problem is solved considering the **equilibrium**:

$$\frac{\partial \sigma_{ij}^m}{\partial x_j} = 0$$

Applying the **principle of virtual power**, the weak form of this local momentum balance equation is

$$\int_{\Omega} \sigma_{ij}^m \frac{\partial v_i^{*m}}{\partial x_j} \, d\Omega = \int_{\Gamma^{int,+}} T_i^+ \, v_i^{*m} \, d\Gamma + \int_{\Gamma^{int,-}} T_i^- \, v_i^{*m} \, d\Gamma$$

where Γ^{int} are internal boundaries since the REV is constituted of blocks and interfaces, T_i are the global components of the interface forces and v_i^{*m} is an admissible virtual velocity field.

⇨ Solved using a **Newton-Raphson** iterative scheme by a linearization of the problem after a spatial discretization with finite elements.

The global (assembling matrix and interface) mechanical stiffness matrix $[K_{mm}]$ yields the incremental relation between the nodal displacement $\{\delta u\}$ and the nodal force $\{\delta f\}$:

$$[K_{mm}]\{\delta u\} = \{\delta f\} \qquad \text{(valid for a given distribution of the fluid pressures)}$$

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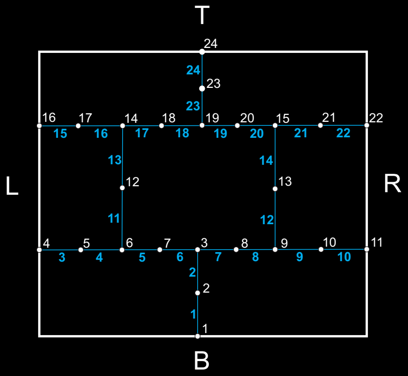
2 The hydraulic problem is solved for a given mechanical configuration.

Hydraulic network:



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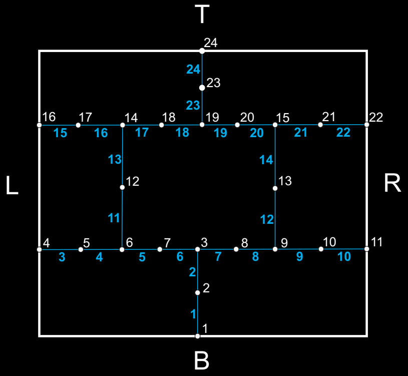


Anti-symmetric boundary fluxes:

$$\omega^3 + \omega^{15} = -(\omega^{10} + \omega^{22}) \qquad \omega^1 = -\omega^{24}$$

Macroscopic pressure gradient:

$$p_{11} - p_4 = (\nabla p)_x^M \ \Delta x \qquad p_{24} - p_1 = (\nabla p)_y^M \ \Delta y$$

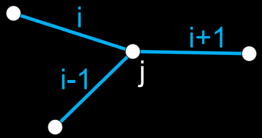
$$p_{22} - p_{16} = (\nabla p)_x^M \ \Delta x$$

Macroscopic pressure:

$$p_k = p^M$$

The microscale problem is solved under **steady-state conditions**.

How is the hydraulic problem established?



Mass balance on node j:

$$\omega^{i-1} + \omega^{i} + \omega^{i+1} = 0$$

with $\omega^{i} = \phi^{i}\left(p_j - p_{j-1}\right)$

$\phi \equiv$ Channel flow model

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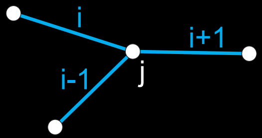

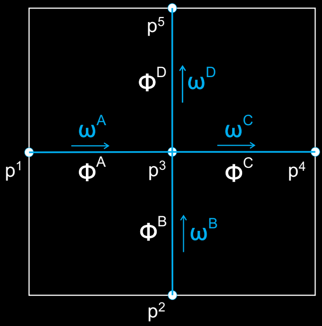

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E.g. solve the linear system:

$$\begin{bmatrix} \phi^A & 0 & -\phi^A - \phi^C & \phi^C & 0 \\ 0 & \phi^B & -\phi^B - \phi^D & 0 & \phi^D \\ -\phi^A & -\phi^B & \phi^A + \phi^B + \phi^C + \phi^D & -\phi^C & -\phi^D \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ (\Delta p)_x^M \\ (\Delta p)_y^M \end{pmatrix}$$

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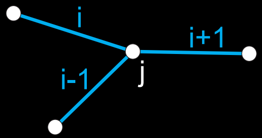

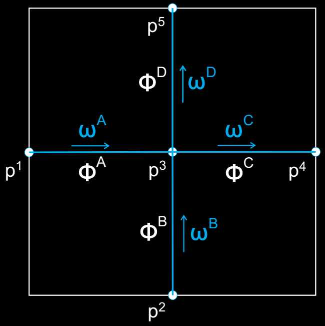

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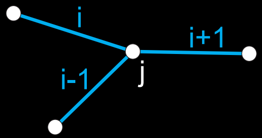

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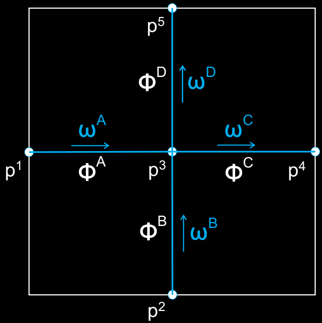

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The **water** pressure system is **first** solved (independent from micro gas pressures) and **then** the **gas** pressure system (dependent from micro water pressures **due to dissolved gas**).

3 Microscale variables are finally homogenized to be sent to the macroscale.

The micro-to-macro transition is derived from the **Hill-Mandel** macro-homogeneity condition:

$$\boxed{\text{Average microscale work = macroscale work}}$$

[Hill, 1965] & [Mandel, 1972]

1) It is satisfied with:

$$\sigma_{ij}^{M} = \frac{1}{\Omega}\int_{\Omega} \sigma_{ij}^{m}\, d\Omega$$

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1) Stresses

$$\sigma_{ij}^{M} = \frac{1}{\Omega} \int_{\Omega} \sigma_{ij}^{m} \, d\Omega$$

2) Given the steady state conditions at the microscale,

the macroscale fluxes are the integrals of the microscale boundary fluxes:

$$f_i^{M} = \frac{1}{\Omega} \int_{\Gamma} \overline{q}^{m} \, x_i \, d\Gamma$$

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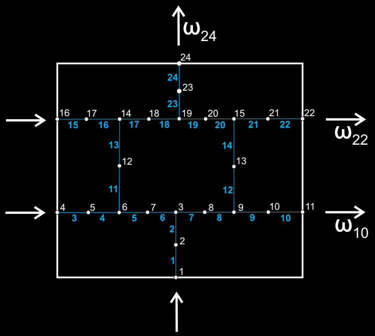

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For instance, the **homogenized fluxes** are

$$f_x^{M} = \omega_{10} + \omega_{22}$$
$$f_y^{M} = \omega_{24}$$

3 Microscale variables are finally homogenized to be sent to the macroscale.

1) Stresses

$$\sigma_{ij}^M = \frac{1}{\Omega}\int_{\Omega} \sigma_{ij}^m \, d\Omega$$

2) Fluid fluxes

$$f_i^M = \frac{1}{\Omega}\int_{\Gamma} \bar{q}^m \, x_i \, d\Gamma$$

3) The macroscopic fluid contents are the total amounts of fluid in the REV:

$$M_w^M = \frac{1}{\Omega}\int_{\Omega_w^{int}} \rho_w \, d\Omega = \rho_w \, S_r \, \phi_f$$

$$M_g^M = M_{g,f}^g + M_{g,f}^d = \rho_g \, (1 - S_r) \, \phi_f + \rho_g^d \, S_r \, \phi_f$$

Finally, the fluid mass storage terms are

$$\dot{M}^{M,t} \approx \frac{M^{M,t} - M^{M,t-\Delta t}}{\Delta t}$$

4. The macroscale problem is also solved using finite elements $\rightarrow$ FE^2

The macroscale computations are governed by

$$\begin{bmatrix} [K^M_{mm}]_{(4x4)} & [K^M_{mw}]_{(4x3)} & [K^M_{mg}]_{(4x3)} \\ [K^M_{wm}]_{(3x4)} & [K^M_{ww}]_{(3x3)} & [K^M_{wg}]_{(3x3)} \\ [K^M_{gm}]_{(3x4)} & [K^M_{gw}]_{(3x3)} & [K^M_{gg}]_{(3x3)} \end{bmatrix} \left\{ \begin{array}{c} \{\delta\varepsilon^M\}_{(4)} \\ \left\{ \begin{array}{c} \delta\nabla p^M_w \\ \delta p^M_w \end{array} \right\}_{(3)} \\ \left\{ \begin{array}{c} \delta\nabla p^M_g \\ \delta p^M_g \end{array} \right\}_{(3)} \end{array} \right\} = \left\{ \begin{array}{c} \{\delta\sigma^M\}_{(4)} \\ \left\{ \begin{array}{c} \delta f^M_w \\ \delta \dot{M}^M_w \end{array} \right\}_{(3)} \\ \left\{ \begin{array}{c} \delta f^M_g \\ \delta \dot{M}^M_g \end{array} \right\}_{(3)} \end{array} \right\}$$

which can be summarized as

$$[A^M]_{(10x10)}\{\delta U^M\}_{(10)} = \{\delta\Sigma^M\}_{(10)}$$

where $[A^M]$ is the macroscale stiffness matrix, $\{\delta U^M\}$ contains the infinitesimal variations of the macroscale variables and $\{\delta\Sigma^M\}$ is their responses.

This stiffness matrix $[A^M]$ is obtained by numerical perturbations.

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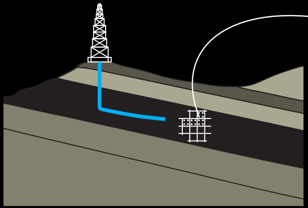
Summary of the *FE*^{2} algorithm

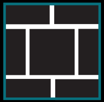
1. **Macroscopic structure** discretised by finite elements

2. **Macroscopic deformation gradient tensor** computed for each IP from the estimation of the macroscopic nodal displacements relative to the external load

3. **REV** assigned at each macroscopic IP



4. **Localization**: apply appropriate **displacements to the REV** from the macroscopic deformation gradient tensor



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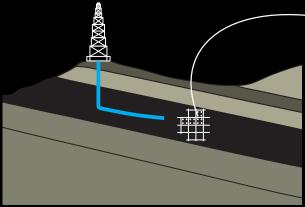

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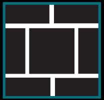


5. **Microscale FE computation**:
stress and deformation distributions in the REV

I. Initial configuration
(nodes coordinates, stress, interfaces aperture)

II. Boundary conditions application

III. Newton-Raphson iterative loop

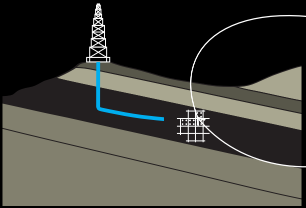
a) Interface apertures
b) Fluid problem solving
c) Interfaces forces
d) Mechanical problem solving
e) Coordinates update
f) Check convergence

Summary of the FE^2 algorithm

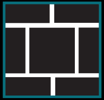
1. **Macroscopic structure** discretised by finite elements

2. **Macroscopic deformation gradient tensor** computed for each IP from the estimation of the macroscopic nodal displacements relative to the external load

3. **REV** assigned at each macroscopic IP



4. **Localization**: apply appropriate **displacements to the REV** from the macroscopic deformation gradient tensor



5. **Microscale FE computation**:
stress and deformation distributions in the REV

I. Initial configuration
(nodes coordinates, stress, interfaces aperture)

II. Boundary conditions application

III. Newton-Raphson iterative loop

a) Interface apertures
b) Fluid problem solving
c) Interfaces forces
d) Mechanical problem solving
e) Coordinates update
f) Check convergence

\-

\+

6. **Homogenization**: REV **averaged stress** returned to the macroscopic IP

7. Macroscopic **internal nodal forces**

8. **Macroscopic stiffness matrix**

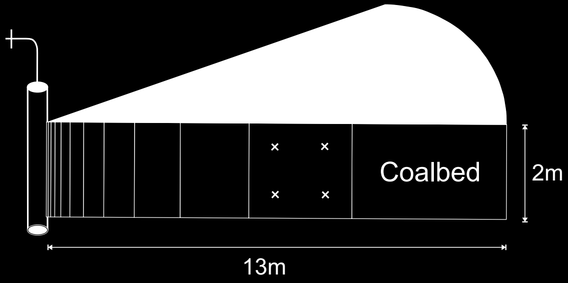
9. **Balance** between external load and internal load?

\+ **Next time step** increment evaluated

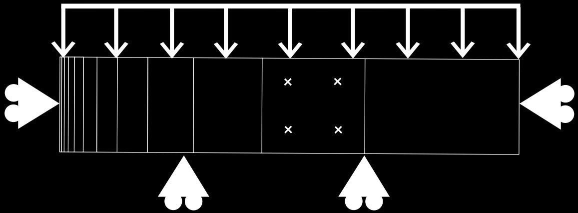
\- **Updated estimation** of the nodal **displacements** required (via macroscopic stiffness matrix)

Lagamine in action

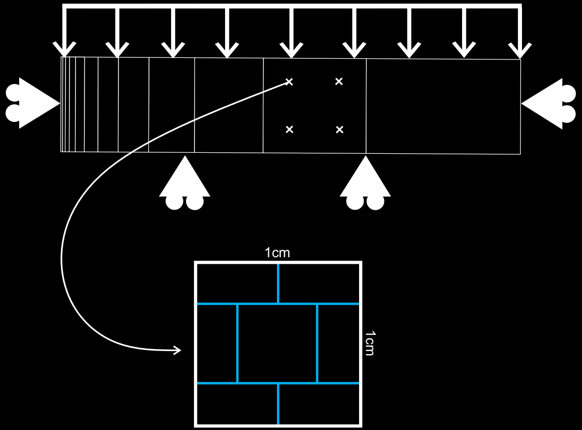
Statement of the problem



Statement of the problem: Boundary conditions



Statement of the problem: Representative Elementary Volume



REV with few blocks in staggered rows

In practice in *.lag* :

```

     5    3   66    3   27    0    0    0    0    3    0    0    0    0    1
     1                90000              1
                         93   72   24   24

NODES
     1         0        -1     3e+06     3e+06
     2         0         0     3e+06     3e+06
     3         0         1     3e+06     3e+06
     4     0.025        -1     3e+06     3e+06
     5     0.025         1     3e+06     3e+06
     6      0.05        -1     3e+06     3e+06
     7      0.05         0     3e+06     3e+06
     8      0.05         1     3e+06     3e+06
     .
     .
     .
    61   12.8746        -1     3e+06     3e+06
    62   12.8746         0     3e+06     3e+06
    63   12.8746         1     3e+06     3e+06
    64         0        -1     3e+06     3e+06
    65         0         0     3e+06     3e+06
    66         0         1     3e+06     3e+06
           Microstructure
     10.0000E+000.0000E+00
     20.1000E+000.0000E+00
     30.2000E+000.0000E+00
     .
     .
     .
    910.8000E+000.1000E+01
    920.9000E+000.1000E+01
    930.1000E+010.1000E+01
                  Corners
     1        10        84        93
RENUM
   210         2         2
FIXED
     1    1  -66
     2    1  -66
     3   64  -66
```

In practice in *.lag* :

```

    5    3   66    3   27    0    0    0    0    3    0    0    0    0    1
    1                90000                1
                        93   72   24   24

NODES
    1         0         -1      3e+06      3e+06
    2         0          0      3e+06      3e+06
    3         0          1      3e+06      3e+06
    4     0.025         -1      3e+06      3e+06
    5     0.025          1      3e+06      3e+06
    6      0.05         -1      3e+06      3e+06
    7      0.05          0      3e+06      3e+06
    8      0.05          1      3e+06      3e+06
    .
    .
    .
   61   12.8746         -1      3e+06      3e+06
   62   12.8746          0      3e+06      3e+06
   63   12.8746          1      3e+06      3e+06
   64         0         -1      3e+06      3e+06
   65         0          0      3e+06      3e+06
   66         0          1      3e+06      3e+06
           Microstructure
    10.0000E+000.0000E+00
    20.1000E+000.0000E+00
    30.2000E+000.0000E+00
    .
    .
    .
   910.8000E+000.1000E+01
   920.9000E+000.1000E+01
   930.1000E+010.1000E+01
                  Corners
    1        10         84         93
RENUM
  210         2          2
FIXED
    1    1  -66
    2    1  -66
    3   64  -66
```

IFEM2

In practice in *.lag* :

```

    5    3   66    3   27    0    0    0    0    3    0    0    0    0    1
    1                90000               1
                        93   72   24   24

NODES
    1         0        -1      3e+06      3e+06
    2         0         0      3e+06      3e+06
    3         0         1      3e+06      3e+06
    4     0.025        -1      3e+06      3e+06
    5     0.025         1      3e+06      3e+06
    6      0.05        -1      3e+06      3e+06
    7      0.05         0      3e+06      3e+06
    8      0.05         1      3e+06      3e+06
    .
    .
    .
   61   12.8746        -1      3e+06      3e+06
   62   12.8746         0      3e+06      3e+06
   63   12.8746         1      3e+06      3e+06
   64         0        -1      3e+06      3e+06
   65         0         0      3e+06      3e+06
   66         0         1      3e+06      3e+06
           Microstructure
    10.0000E+000.0000E+00
    20.1000E+000.0000E+00
    30.2000E+000.0000E+00
    .
    .
    .
   910.8000E+000.1000E+01
   920.9000E+000.1000E+01
   930.1000E+010.1000E+01
                  Corners
    1        10        84        93
RENUM
  210         2         2
FIXED
    1    1  -66
    2    1  -66
    3   64  -66
```

IFEM2

NUMNP2
NUMEL2
NUMNDH
NUMELH

In practice in *.lag* :

```
     5    3   66    3   27    0    0    0    0    3    0    0    0    0    1
     1                90000                   1
                        93   72   24   24

NODES
     1         0        -1      3e+06      3e+06
     2         0         0      3e+06      3e+06
     3         0         1      3e+06      3e+06
     4     0.025        -1      3e+06      3e+06
     5     0.025         1      3e+06      3e+06
     6      0.05        -1      3e+06      3e+06
     7      0.05         0      3e+06      3e+06
     8      0.05         1      3e+06      3e+06
     .
     .
     .
    61   12.8746        -1      3e+06      3e+06
    62   12.8746         0      3e+06      3e+06
    63   12.8746         1      3e+06      3e+06
    64         0        -1      3e+06      3e+06
    65         0         0      3e+06      3e+06
    66         0         1      3e+06      3e+06
           Microstructure
     10.0000E+000.0000E+00
     20.1000E+000.0000E+00
     30.2000E+000.0000E+00
     .
     .
     .
    910.8000E+000.1000E+01
    920.9000E+000.1000E+01
    930.1000E+010.1000E+01
                   Corners
     1        10        84        93
RENUM
   210         2         2
FIXED
     1    1  -66
     2    1  -66
     3   64  -66
```

IFEM2

NUMNP2
NUMEL2
NUMNDH
NUMELH

Definition of the microstructure
(93 nodes)

Nodes

In practice in *.lag* :

```
RENUM
  210         2         2
FIXED
    1    1  -66
    2    1  -66
    3   64  -66
    5    1  -66

FORCE

COLAW
    1  625          *** FE2 ****
         3         4       100         2         3         1
         1         1         1
       1.0      0.25      1E-5       1.0   1.0E-06  2.00E-05   1.0E-02
   1.0E-03   1.0E+03   5.0E-10   3.0E+06    293.15
   1.1E-05  20.88E-0 16.04E-03   3.0E+06    0.0347  1.84E-09
    2.0E-2     1.5E6      1.00     3.6E4
       1E5      0.25       0.1       0.0     1E-05
    1    1    1
    1.5E03    1.21E9      0.16      0.40
    2    0    1
  1.00E11   1.00E11      2.00  2.00E-05
    3    0    1
  1.00E11   1.00E11      2.00  2.00E-05
    2   95          *** LICHA ****
    1    0
   -5.0E06       0.0       0.0       0.0       0.0       0.0
    3  194          *** WPROG ****
   23   13    8    0
  1.00E-10
   1.0E-03   1.0E+03   1.1E-05   6.58E-0   3.0E+06   3.0E+06    0.0347   5.0E-10
      0.10        0.        0.        0.      0.25        0.       1E5        0.
       3.0       0.5       2.0
       3.0        1.        0.
       0.0
ELFE2
   12    1
-5.00000E6        0.        1.        0.
    8    4    1    8    4    0
    1    4    6    7    8    5    3    2
```

Laws

In practice in *.lag* :

```
RENUM
  210         2         2
FIXED
    1    1  -66
    2    1  -66
    3   64  -66
    5    1  -66

FORCE

COLAW
    1  625          *** FE2 ****
         3         4       100         2         3         1
         1         1         1
       1.0      0.25      1E-5       1.0   1.0E-06  2.00E-05   1.0E-02
   1.0E-03   1.0E+03  5.0E-10   3.0E+06    293.15
   1.1E-05  20.88E-0 16.04E-03   3.0E+06    0.0347  1.84E-09
    2.0E-2     1.5E6      1.00     3.6E4
       1E5      0.25       0.1       0.0     1E-05
    1    1    1
    1.5E03    1.21E9      0.16      0.40
    2    0    1
   1.00E11   1.00E11      2.00  2.00E-05
    3    0    1
   1.00E11   1.00E11      2.00  2.00E-05
    2   95          *** LICHA ****
    1    0
   -5.0E06       0.0       0.0       0.0       0.0       0.0
    3  194          *** WPROG ****
   23   13    8    0
  1.00E-10
   1.0E-03   1.0E+03   1.1E-05   6.58E-0   3.0E+06   3.0E+06    0.0347   5.0E-10
      0.10        0.        0.        0.      0.25        0.       1E5        0.
       3.0       0.5       2.0
       3.0        1.        0.
       0.0
ELFE2
   12    1
-5.00000E6        0.        1.        0.
    8    4    1    8    4    0
    1    4    6    7    8    5    3    2
```

FE2 law
defined as a usual law

Laws

In practice in *.lag* :

```
RENUM
  210        2        2
FIXED
    1    1  -66
    2    1  -66
    3   64  -66
    5    1  -66

FORCE

COLAW
    1  625          *** FE2 ****
         3         4       100         2         3         1
         1         1         1
       1.0      0.25      1E-5       1.0   1.0E-06  2.00E-05   1.0E-02
   1.0E-03   1.0E+03   5.0E-10   3.0E+06    293.15
   1.1E-05  20.88E-0 16.04E-03   3.0E+06    0.0347  1.84E-09
    2.0E-2     1.5E6      1.00     3.6E4
       1E5      0.25       0.1       0.0     1E-05
    1    1    1
    1.5E03    1.21E9      0.16      0.40
    2    0    1
  1.00E11   1.00E11       2.00  2.00E-05
    3    0    1
  1.00E11   1.00E11       2.00  2.00E-05
    2   95          *** LICHA ****
    1    0
   -5.0E06       0.0       0.0       0.0       0.0       0.0
    3  194          *** WPROG ****
   23   13    8    0
  1.00E-10
   1.0E-03   1.0E+03   1.1E-05   6.58E-0   3.0E+06   3.0E+06    0.0347   5.0E-10
      0.10        0.        0.        0.      0.25        0.       1E5        0.
       3.0       0.5       2.0
       3.0        1.        0.
       0.0
ELFE2
   12    1
-5.00000E6        0.        1.        0.
    8    4    1    8    4    0
    1    4    6    7    8    5    3    2
```

FE2 law
defined as a usual law

Micro-laws
inside the law

Laws

In practice in *.lag* :

Elements

```
          0.0
ELFE2
   12    1
-5.00000E6        0.        1.        0.
    8    4    1    8    4    0
    1    4    6    7    8    5    3    2
    8    4    1    8    4    0
    6    9   11   12   13   10    8    7
    .
    .
    .
    8    4    1    8    4    0
   56   59   61   62   63   60   58   57
   72
    1    1    1    2   12   11
    2    1    2    3   13   12
    3    1    3    4   14   13
    .
    .
    .
   70    1   80   81   91   90
   71    1   81   82   92   91
   72    1   82   83   93   92
   24
    1    1    1    0    0
    2    0    1    2    0
    3    0    2    6    7
    .
    .
    .
   23    0   23   24    0
   24    2   24    0    0
   24
    1    5    2    1    0
    2   14    3    2    0
    .
    .
    .
   23   59   23   19    0
   24   68   24   23    0
LICHA
```

In practice in *.lag* :

```
        0.0
ELFE2
   12    1
-5.00000E6        0.        1.        0.
    8    4    1    8    4    0
    1    4    6    7    8    5    3    2
    8    4    1    8    4    0
    6    9   11   12   13   10    8    7
    .
    .
    .
    8    4    1    8    4    0
   56   59   61   62   63   60   58   57
   72
    1    1    1    2   12   11
    2    1    2    3   13   12
    3    1    3    4   14   13
    .
    .
    .
   70    1   80   81   91   90
   71    1   81   82   92   91
   72    1   82   83   93   92
   24
    1    1    1    0    0
    2    0    1    2    0
    3    0    2    6    7
    .
    .
    .
   23    0   23   24    0
   24    2   24    0    0
   24
    1    5    2    1    0
    2   14    3    2    0
    .
    .
    .
   23   59   23   19    0
   24   68   24   23    0
LICHA
```

Macro-elements
(12)

Elements

In practice in *.lag* :

Elements

```
        0.0
ELFE2
   12    1
-5.00000E6        0.        1.        0.
    8    4    1    8    4    0
    1    4    6    7    8    5    3    2
    8    4    1    8    4    0
    6    9   11   12   13   10    8    7
    .
    .
    .
    8    4    1    8    4    0
   56   59   61   62   63   60   58   57
   72
    1    1    1    2   12   11
    2    1    2    3   13   12
    3    1    3    4   14   13
    .
    .
    .
   70    1   80   81   91   90
   71    1   81   82   92   91
   72    1   82   83   93   92
   24
    1    1    1    0    0
    2    0    1    2    0
    3    0    2    6    7
    .
    .
    .
   23    0   23   24    0
   24    2   24    0    0
   24
    1    5    2    1    0
    2   14    3    2    0
    .
    .
    .
   23   59   23   19    0
   24   68   24   23    0
LICHA
```

Macro-elements (12)

Micro-elements (72)

In practice in *.lag* :

```
          0.0
ELFE2
   12    1
-5.00000E6        0.        1.        0.
    8    4    1    8    4    0
    1    4    6    7    8    5    3    2
    8    4    1    8    4    0
    6    9   11   12   13   10    8    7
    .
    .
    .
    8    4    1    8    4    0
   56   59   61   62   63   60   58   57
   72
    1    1    1    2   12   11
    2    1    2    3   13   12
    3    1    3    4   14   13
    .
    .
    .
   70    1   80   81   91   90
   71    1   81   82   92   91
   72    1   82   83   93   92
   24
    1    1    1    0    0
    2    0    1    2    0
    3    0    2    6    7
    .
    .
    .
   23    0   23   24    0
   24    2   24    0    0
   24
    1    5    2    1    0
    2   14    3    2    0
    .
    .
    .
   23   59   23   19    0
   24   68   24   23    0
LICHA
```

Macro-elements (12)

Micro-elements (72)

Nodes of the hydraulic network (24)

Elements

In practice in *.lag* :

Elements

```
          0.0
ELFE2
   12    1
-5.00000E6        0.        1.        0.
    8    4    1    8    4    0
    1    4    6    7    8    5    3    2
    8    4    1    8    4    0
    6    9   11   12   13   10    8    7
    .
    .
    .
    8    4    1    8    4    0
   56   59   61   62   63   60   58   57
   72
    1    1    1    2   12   11
    2    1    2    3   13   12
    3    1    3    4   14   13
    .
    .
    .
   70    1   80   81   91   90
   71    1   81   82   92   91
   72    1   82   83   93   92
   24
    1    1    1    0    0
    2    0    1    2    0
    3    0    2    6    7
    .
    .
    .
   23    0   23   24    0
   24    2   24    0    0
   24
    1    5    2    1    0
    2   14    3    2    0
    .
    .
    .
   23   59   23   19    0
   24   68   24   23    0
LICHA
```

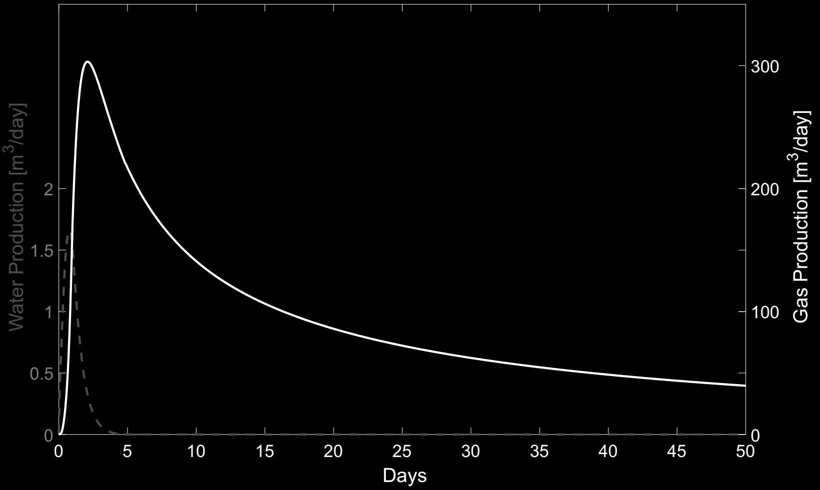
Macro-elements (12)

Micro-elements (72)

Nodes of the hydraulic network (24)

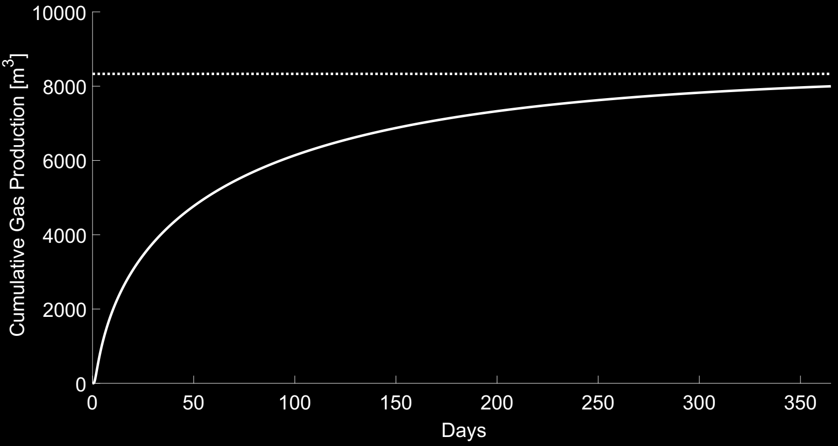
Elements of the hydraulic network (24)

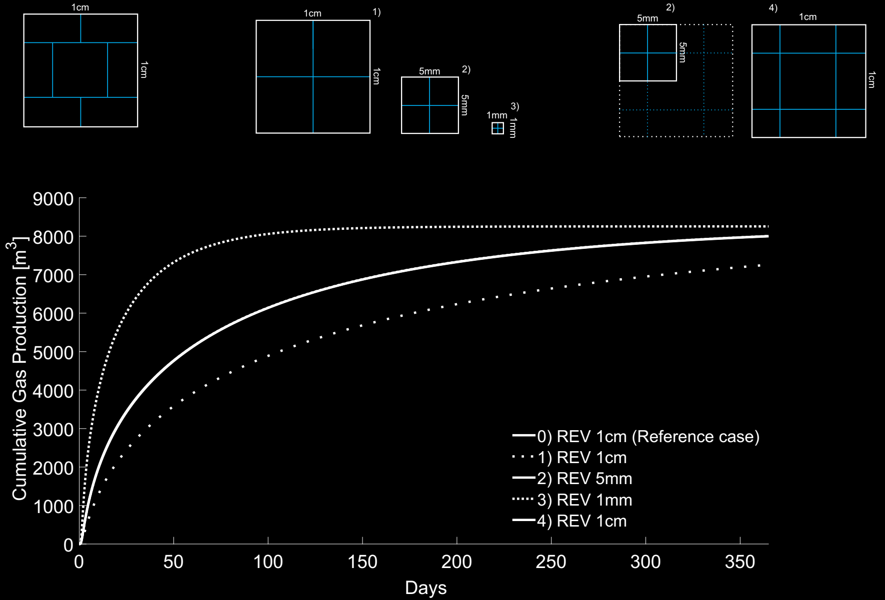
A reference set of parameters is chosen and gives the following production curves.



The reservoir first desaturates and then gas production peaks.

The total production tends to the quantity of gas initially stored (fortunately!).




1cm
1cm
1)
1cm
1cm
2)
5mm
5mm
3)
1mm
1mm
2)
5mm
5mm
4)
1cm
1cm
Cumulative Gas Production [m^3]
Days
0) REV 1cm (Reference case)
1) REV 1cm
2) REV 5mm
3) REV 1mm
4) REV 1cm

Conclusions

In **summary**,

- Microscale model (fracture-scale) [Bertrand et al., 2019]
  It is highly accurate but computationally expensive (impossible for the reservoir scale).

- Macroscale model (homogenized laws) [Bertrand et al., 2017]
  It is suitable for reservoir modelling but less flexible.

- Multiscale model (FE^2) [Paper in preparation]
  It is the compromise solution for reservoir modelling.

In **summary**,

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In **summary**,

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  It is suitable for reservoir modelling but less flexible.

- Multiscale model (FE^2) [Paper in preparation]
  It is the compromise solution for reservoir modelling.

Developments in the Lagamine code:

- Multiscale model
    - A complete reorganization of the input file.
    - A new degree of freedom for gas with *ad hoc* unsaturated laws.
    - The consideration of an adsorbed gas with the resulting couplings.
    - The implementation of a new mechanical law for the interface (Bandis type).
    - The possibility to change the REV size.
    - The definition of initial stresses.
    - The plane strain implementation for the REV (instead of true 2D) combined with axisymmetric conditions for the reservoir scale.

The FE^2 method

Multiscale hydro-mechanical modelling in the Lagamine code

Researches supported by the FNRS - FRIA and the WBI

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International.be

The FE^2 method

Multiscale hydro-mechanical modelling in the Lagamine code

Researches supported by the FNRS - FRIA and the WBI

References

References I

Bertrand, F., Buzzi, O., and Collin, F. (2019).
Cleat-scale modelling of the coal permeability evolution due to sorption-induced strain.
International Journal of Coal Geology.

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